Your Signature ____________________

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Additional sheets taken, if any, should be properly attached to the main answer booklet.

3. **Please write your name on every page of this booklet and every additional sheet taken.**

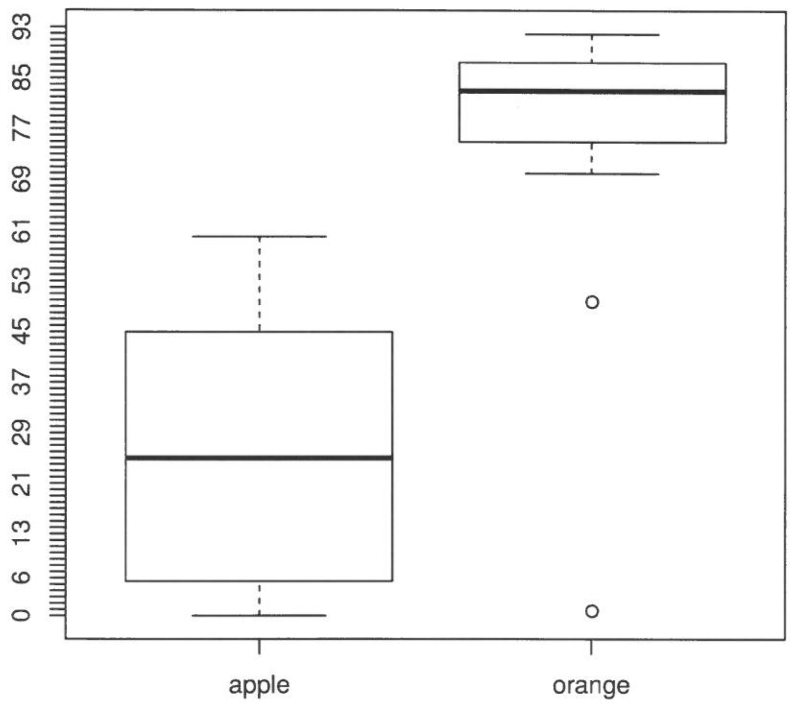
Score

Q.No.	Alloted Score	Score
1.	14	
2.	20	
3.	20	
4.	16	
5.	15	
6.	15	
Total	100	

Number of Extra sheets attached to the answer script: ____________________

1. (a) Below there are two box plots that are shown.

Comparison of weights in Pounds



(i) Which fruit has a higher median weight ?

(ii) What is the approximate median weight of oranges ?

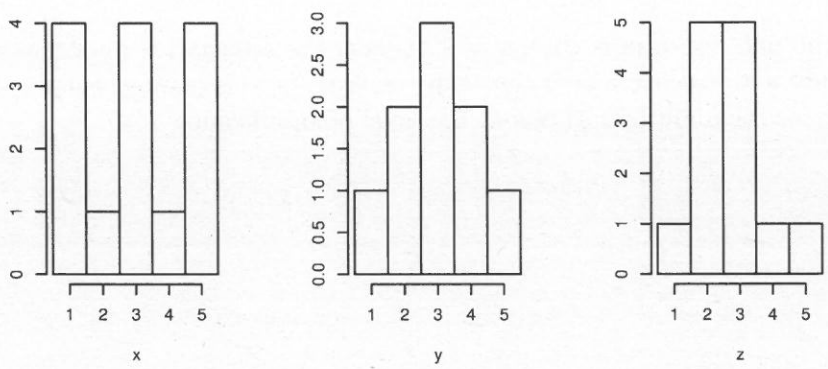
(iii) What is the approximate `IQR` for appless ?

(iv) How many outliers are there in this data set ?

(v) Which fruit has a larger range of weights ?

1(b) Please circle the correct choice. No justification is required.

Consider the three histograms given below of datasets x, y and z



The ordering of the dataset from the smallest to biggest standard deviations is given by:

$(i)(\mathrm{x}, \mathrm{y}, \mathrm{z})$ $(ii)(\mathrm{x}, \mathrm{z}, \mathrm{y})$ $(iii)(\mathrm{y}, \mathrm{x}, \mathrm{z})$ $(iv)(\mathrm{y}, \mathrm{z}, \mathrm{x})$ $(v)(\mathrm{z}, \mathrm{y}, \mathrm{x})$ $(vi)(\mathrm{z}, \mathrm{x}, \mathrm{y})$

2. Suppose data $X_1, X_2, \ldots X_n$ are i.i.d. and drawn from Normal(μ, σ^2), where μ and σ are unknown. Suppose we have $n = 49$, sample mean is 92 and sample standard deviation is 0.75.

(a) Compute a 95% confidence interval for the mean, μ.

(b) We want to test the null hypothesis that $\mu = 90$ versus the alternative hypothesis that $\mu > 90$. Decide and execute a test that can check if there is enough evidence whether one can reject the null hypothesis at 5% level of significance.

3. Let $\mu \in \mathbb{R}$ and $\sigma > 0$ and $(X_1, X_2, \ldots, X_n)$ be from a population distributed as Normal with mean μ and variance σ^2. Then find the M.L.E. for μ, σ.

4. Let $g : \mathbb{R} \to [0, \infty)$ and $f : \mathbb{R} \to [0, \infty)$ are two probability density functions such that

$$f(x) \leq g(x), \quad \forall x \in \mathbb{R}.$$

Consider the following Acceptance-Rejection Algorithm:-

(a) Generate a sample Y having density g.

(b) Generate a sample U from Uniform$(0, 1)$.

(c) If $U \leq \frac{f(Y)}{g(Y)}$ then set $X = Y$, else repeat step (a).

1. Show that X has probability density function f.
2. Using the Acceptance-Rejection Algorithm described above, write a function in `R`, called `mybeta` to generate sample of Beta(2,4).

5. The student body at Hello University consists of 20% Master's students, 24% third year students, 26% second year students, and 30% first year students. Suppose a researcher takes a sample of 50 such students. Within the sample there are 13 Master's, 16 Third year's, 10 Second year's, and 11 First year's. The researcher claims that his sampling procedure should have produced independent selections from the student body, with each student equally likely to be selected. Is this a plausible claim given the observed results?

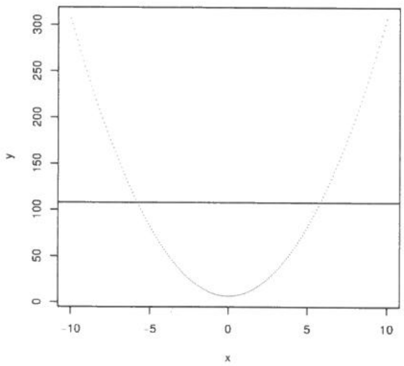
6. Consider the following R commands.

```
> x = seq(-10,10,by=1/10)
> y = 3*x^2 - 0.0001*x +7
> lm(y~x)


Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)            x
   108.0000      -0.0001
```

```
> plot(y~x,pch=16, cex=0.2, col="red")
> abline(lm(y~x), col="blue")
```



(a) Explain what is being executed in the above R code.
(b) Explain the reasons behind the straight line in contrast to the quadratic curve.

Statistical Tables for Reference if Needed.

Table 5: in (i,j)-th entry provides $\mathbb{P}(\chi_i^2 \le j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
2	0.181	0.221	0.259	0.295	0.330	0.362	0.393	0.423	0.451	0.478	0.503
3	0.060	0.081	0.104	0.127	0.151	0.175	0.199	0.223	0.247	0.271	0.294
4	0.018	0.026	0.037	0.049	0.062	0.075	0.090	0.106	0.122	0.139	0.156
5	0.005	0.008	0.012	0.017	0.023	0.030	0.037	0.046	0.055	0.065	0.076
6	0.001	0.002	0.004	0.006	0.008	0.011	0.014	0.018	0.023	0.028	0.034
7	0.000	0.001	0.001	0.002	0.003	0.004	0.005	0.007	0.009	0.012	0.014
8	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.006
9	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002

Table 6: in (i,j)-th entry provides $\mathbb{P}(\chi_i^2 \le j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
2	0.528	0.551	0.573	0.593	0.613	0.632	0.650	0.667	0.683	0.699	0.713
3	0.318	0.341	0.363	0.385	0.407	0.428	0.448	0.468	0.487	0.506	0.525
4	0.173	0.191	0.209	0.228	0.246	0.264	0.283	0.301	0.319	0.337	0.355
5	0.087	0.099	0.111	0.124	0.137	0.151	0.165	0.179	0.194	0.209	0.224
6	0.041	0.047	0.055	0.063	0.071	0.080	0.090	0.100	0.110	0.121	0.132
7	0.018	0.021	0.025	0.030	0.035	0.040	0.046	0.052	0.059	0.066	0.073
8	0.007	0.009	0.011	0.013	0.016	0.019	0.022	0.026	0.030	0.034	0.038
9	0.003	0.004	0.005	0.006	0.007	0.009	0.010	0.012	0.014	0.017	0.019

Table 7: in (i,j)-th entry provides $\mathbb{P}(\chi_i^2 \le j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	2.50	2.52	2.54	2.56	2.58	2.60	2.62	2.64	2.66	2.68	2.70
2	0.713	0.716	0.719	0.722	0.725	0.727	0.730	0.733	0.736	0.738	0.741
3	0.525	0.528	0.532	0.535	0.539	0.543	0.546	0.549	0.553	0.556	0.560
4	0.355	0.359	0.363	0.366	0.370	0.373	0.377	0.380	0.384	0.387	0.391
5	0.224	0.227	0.230	0.233	0.236	0.239	0.242	0.245	0.248	0.251	0.254
6	0.132	0.134	0.136	0.138	0.141	0.143	0.145	0.148	0.150	0.152	0.155
7	0.073	0.074	0.076	0.077	0.079	0.081	0.082	0.084	0.085	0.087	0.089
8	0.038	0.039	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048
9	0.019	0.020	0.020	0.021	0.021	0.022	0.023	0.023	0.024	0.024	0.025

Table 8: in (i,j)-th entry provides $\mathbb{P}(F(2,i) \le j)$, with $F(2,i) \sim$ F-distribution with degrees of freedom 2 and i.

	4.40	4.42	4.44	4.46	4.48	4.50	4.52	4.54	4.56	4.58	4.60
1	0.681	0.681	0.682	0.682	0.683	0.684	0.684	0.685	0.686	0.686	0.687
2	0.815	0.815	0.816	0.817	0.818	0.818	0.819	0.819	0.820	0.821	0.821
3	0.872	0.872	0.873	0.874	0.874	0.875	0.876	0.876	0.877	0.877	0.878
4	0.902	0.903	0.904	0.904	0.905	0.905	0.906	0.906	0.907	0.908	0.908
5	0.921	0.922	0.922	0.923	0.923	0.924	0.924	0.925	0.925	0.926	0.926
6	0.933	0.934	0.934	0.935	0.935	0.936	0.937	0.937	0.938	0.938	0.938
7	0.942	0.943	0.943	0.944	0.944	0.945	0.945	0.946	0.946	0.947	0.947
8	0.949	0.949	0.950	0.950	0.950	0.951	0.951	0.952	0.952	0.953	0.953
9	0.954	0.954	0.954	0.955	0.955	0.956	0.956	0.957	0.957	0.958	0.958
10	0.957	0.958	0.958	0.959	0.959	0.960	0.960	0.960	0.961	0.961	0.962
11	0.961	0.961	0.961	0.962	0.962	0.963	0.963	0.963	0.964	0.964	0.965
12	0.963	0.964	0.964	0.964	0.965	0.965	0.966	0.966	0.966	0.967	0.967
13	0.965	0.966	0.966	0.966	0.967	0.967	0.968	0.968	0.968	0.969	0.969
14	0.967	0.967	0.968	0.968	0.969	0.969	0.969	0.970	0.970	0.971	0.971
15	0.969	0.969	0.969	0.970	0.970	0.971	0.971	0.971	0.972	0.972	0.972

Table 9: in (i,j)-th entry provides $\mathbb{P}(F(12,i) \le j)$, with $F(12,i) \sim$ F-distribution with degrees of freedom 12 and i.

	4.40	4.42	4.44	4.46	4.48	4.50	4.52	4.54	4.56	4.58	4.60
1	0.642	0.643	0.644	0.644	0.645	0.646	0.647	0.647	0.648	0.649	0.649
2	0.800	0.801	0.802	0.802	0.803	0.804	0.805	0.805	0.806	0.807	0.808
3	0.876	0.876	0.877	0.878	0.878	0.879	0.880	0.880	0.881	0.882	0.882
4	0.918	0.918	0.919	0.920	0.920	0.921	0.921	0.922	0.923	0.923	0.924
5	0.943	0.944	0.944	0.945	0.945	0.946	0.946	0.947	0.947	0.948	0.948
6	0.960	0.960	0.961	0.961	0.962	0.962	0.962	0.963	0.963	0.963	0.964
7	0.971	0.971	0.971	0.972	0.972	0.972	0.973	0.973	0.973	0.974	0.974
8	0.978	0.979	0.979	0.979	0.979	0.980	0.980	0.980	0.980	0.981	0.981
9	0.983	0.984	0.984	0.984	0.984	0.985	0.985	0.985	0.985	0.986	0.986
10	0.987	0.987	0.988	0.988	0.988	0.988	0.988	0.989	0.989	0.989	0.989
11	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.991	0.991	0.991	0.992
12	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.993	0.993	0.993
13	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995
14	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.996	0.996	0.996	0.996
15	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.997	0.997

November 11, 2019 **Name (Please Print)** ____________________

Statistical Tables for Reference if Needed.

Table 1 : in (i,j)-th entry provides $\mathbb{P}(t_i \le j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
2	0.570	0.604	0.636	0.667	0.695	0.722	0.746	0.768	0.789	0.807	0.823
3	0.573	0.608	0.642	0.674	0.705	0.733	0.759	0.783	0.804	0.824	0.842
4	0.574	0.610	0.645	0.678	0.710	0.739	0.766	0.790	0.813	0.833	0.852
5	0.575	0.612	0.647	0.681	0.713	0.742	0.770	0.795	0.818	0.839	0.858
6	0.576	0.613	0.648	0.683	0.715	0.745	0.773	0.799	0.822	0.843	0.862
7	0.576	0.614	0.649	0.684	0.716	0.747	0.775	0.801	0.825	0.846	0.865
8	0.577	0.614	0.650	0.685	0.717	0.748	0.777	0.803	0.827	0.848	0.868
9	0.577	0.615	0.651	0.685	0.718	0.749	0.778	0.804	0.828	0.850	0.870

Table 2 : in (i,j)-th entry provides $\mathbb{P}(t_i \le j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
2	0.823	0.838	0.852	0.864	0.875	0.884	0.893	0.901	0.908	0.915	0.921
3	0.842	0.858	0.872	0.885	0.896	0.906	0.915	0.923	0.930	0.937	0.942
4	0.852	0.868	0.883	0.896	0.908	0.918	0.927	0.935	0.942	0.948	0.954
5	0.858	0.875	0.890	0.903	0.915	0.925	0.934	0.942	0.949	0.955	0.960
6	0.862	0.879	0.894	0.908	0.920	0.930	0.939	0.947	0.954	0.960	0.965
7	0.865	0.883	0.898	0.911	0.923	0.934	0.943	0.950	0.957	0.963	0.968
8	0.868	0.885	0.900	0.914	0.926	0.936	0.945	0.953	0.960	0.966	0.971
9	0.870	0.887	0.902	0.916	0.928	0.938	0.947	0.955	0.962	0.967	0.972

Table 3 : in (i,j)-th entry provides $\mathbb{P}(t_i \le j)$, with $t_i \sim$ t-distribution with i degrees of freedom.

	2.20	2.22	2.24	2.26	2.28	2.30	2.32	2.34	2.36	2.38	2.40
2	0.921	0.922	0.923	0.924	0.925	0.926	0.927	0.928	0.929	0.930	0.931
3	0.942	0.943	0.945	0.946	0.947	0.948	0.948	0.949	0.950	0.951	0.952
4	0.954	0.955	0.956	0.957	0.958	0.959	0.959	0.960	0.961	0.962	0.963
5	0.960	0.961	0.962	0.963	0.964	0.965	0.966	0.967	0.968	0.968	0.969
6	0.965	0.966	0.967	0.968	0.969	0.969	0.970	0.971	0.972	0.973	0.973
7	0.968	0.969	0.970	0.971	0.972	0.973	0.973	0.974	0.975	0.976	0.976
8	0.971	0.971	0.972	0.973	0.974	0.975	0.976	0.976	0.977	0.978	0.978
9	0.972	0.973	0.974	0.975	0.976	0.977	0.977	0.978	0.979	0.979	0.980

Table 4: in (i,j)-th entry provides $\mathbb{P}(Z \le i+j)$, with $Z \sim$ Normal$(0,1)$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.9554
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.9641
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.9713
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.9772
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.9821
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0.9861
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.9893
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0.9918
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0.9938
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.9953
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9987
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998